Name:______________________________ **Date:__________________________**

Math Contest Club: Complex Variables part 2

Complex Roots

If a polynomial has **real coefficients**, then *either* all roots are real *or* there are an **even number of non-real complex roots, in conjugate pairs.**

For example, if $5+2i$ is a zero of a polynomial with real coefficients, then $5-2i$ must also be a zero of that polynomial. It is equally true that if $(x-5-2i)$ is a factor then $(x-5+2i)$ is also a factor.

Why is this true? Because when you have a factor with an imaginary part and multiply it by its complex conjugate you get a real result:

$$(x-5-2i)(x-5+2i) = x^2-10x+25-4i^2 = x^2-10x+29$$

If $(x-5-2i)$ was a factor but $(x-5+2i)$ was not, then the polynomial would end up with imaginaries in its coefficients, no matter what the other factors might be. If the polynomial has only real coefficients, then any complex roots must occur in conjugate pairs.

1. Find the product or simplify each of the following:

$(5-2i)i$	$(1-i)^2$	$(3-2i)(7+6i)$
$(-2+6i)(3+4i)$	$\dfrac{3+2i}{4-3i}$	$\dfrac{1+i}{1-i}-\dfrac{1-i}{1+i}$
$\dfrac{1+2i}{2+3i}$	$\dfrac{2-2i}{1+2i}+\dfrac{1+3i}{2-i}$	$(x-2-i)(x-2+i)(x+1)$
$\dfrac{2+3i}{4i}$	$\dfrac{3-2i}{2-i}$	$\dfrac{5+\sqrt{3}i}{5-\sqrt{3}i}$
$\dfrac{2}{1-3i}+\dfrac{5}{1+2i}$	$\dfrac{1}{\cos\theta+i\sin\theta}$	$\dfrac{1}{1+\cos\theta-i\sin\theta}$

2. What is the fifth term of the expansion of $(a+ib)^7$?

3. The roots of $2x^2-3x+c=0$ are imaginary for what value(s) of "c"?

4. In the equation $ax^2+bx+c=0$, $a,b,$ *and* c are real numbers. If $\frac{1}{3}-\frac{2}{3}i$ is a root of this equation, what is the sum of the roots?

5. In which quadrant does the sum of $2+3i$ and $3-5i$ lie in?

6. What are the solutions to the equation? $x^2+9=0$

7. MC: Which equation has roots of $5-2i$ and $5+2i$?

a) $x^2-10x+29=0$ b) $x^2-10x-21=0$ c) $x^2+10x-21=0$ d) $x^2+10x+29=0$

8. MC: Which equation below has roots $5+i$ and $5-i$?

a) $x^2-10x+24=0$ b) $x^2-10x+26=0$ c) $x^2+10x+24=0$ d) $x^2+10x+26=0$

9. If $x=2+i$ is a factor of $x^3-3x^2+x+5=0$, then what are all the other roots?

10. If $\left(\frac{1-i\sqrt{5}}{2}\right)^{10}=a+bi$, then what is *b*?

11. First, if $(a+bi)^2=c+di$, then $\pm(a+bi)=\sqrt{c+di}$. Therefore, solve the following quadratic equation with complex coefficients and completely simplify: $z^2+(2-i)z-(1-5i)=0$

12. Solve the following over the complex numbers. Express your answer in $a+bi$ form: $z^2-16-16i\sqrt{3}=0$

13. Consider the following product of two complex numbers, each of which is written in polar trigonometric form. Find the product and write the answer in rectangular form, $a+bi$, where “a” and “b” are real numbers: $6(\cos 75°+i\sin 75°)5(\cos 165°+i\sin 165°)$

14. If two roots of the polynomial: $x^6 - 6x^5 + 14x^4 - 22x^3 + 25x^2 + 8x = 60$ are $2i$ and $2 - i$, then what are the real roots?

15. Let r_1 and r_2 be the roots of the equation $x^2 + bx + c = 0$. If $r_2 - r_1 = 6i$ and $r_1 \times r_2 = 2$, then what is the sum of "b" and "c"?

16. For how many positive integers "n" less than or equal to 1000 is $\left(\sin t + i\cos t\right)^n = \sin\left(nt\right) + i\cos\left(nt\right)$ true for all real "t"? 2005 AIME II

17. Let $F\left(z\right) = \dfrac{z+i}{z-i}$ for all complex numbers $z \neq i$, and let $z_n = F\left(z_{n-1}\right)$ for all positive integers "n". Given that $z_0 = \dfrac{1}{137} + i$ and $z_{2002} = a + bi$, where "a' and "b" are real numbers, find $a + b$ (AIME I 2002)

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23. The solutions of the equation $z^4 + 4z^3 i - 6z^2 - 4zi - i = 0$ are the vertices of a convex polygon in the complex plane. What is the area of the polygon?

(A) $2^{\frac{5}{8}}$ **(B)** $2^{\frac{3}{4}}$ **(C)** 2 **(D)** $2^{\frac{5}{4}}$ **(E)** $2^{\frac{3}{2}}$